Chapter 3.

Simple interest.

Investing money.

Suppose we deposit \$500 into a savings account that earns interest at the rate of 10% per year ("per year" is often referred to as *per annum*). How much will this account be worth after 1 year? How much will this account be worth after 2 years? Suppose we take the money out of the account after just six months, or just a few days?

To answer these questions we would need to know how the account operates and, in particular, how the interest is calculated and when it is added to the account. In this chapter we will consider the process called "simple interest" and in the next chapter we will consider the "compound interest" process.

Simple Interest.

Suppose the \$500 is invested in an account for which the interest is paid at 10% per annum simple interest. This means that each year, 10% of \$500 is added to the account as interest.

Thus after 1 year the account will be worth
\$500 + 10% of \$500
= \$500 + \$50
= \$550

After 2 years the account will be worth
\$500 + 2 × 10% of \$500
= \$500 + 2 × \$50
= \$600

After 3 years the account will be worth
\$500 + 3 × 10% of \$500
= \$500 + 3 × \$50
= \$650

Each year the account is worth \$50 more than the previous year:

Value at end of one year = Value at start of that year + \$50

If we want the value after n years:

Value after n years = Initial value + n × \$50

Notice that whilst the interest is earned each year it is not added to the account until some later date (e.g. when the account is closed). In this way the interest from the first year does not itself earn interest in the second year. This is what distinguishes *simple interest,* considered in this chapter, with *compound interest* considered in the next chapter. In compound interest, interest is earned on the interest.

A spreadsheet could be used to determine and display the value of the account after 1, 2, 3, ... years, as shown on the next page.

	A	B	C	D	E	F	G	H
1	Amount invested ($)		**$500.00**					
2	Annual interest rate		**10.00%**					
3		Simple interest account.						
4	End of year	1	$550.00					
5		2	$600.00					
6		3	$650.00					
7		4	$700.00					
8		5	$750.00					
9		6	$800.00					
10		7	$850.00					
11		8	$900.00					

Try to create such a spreadsheet for which you can simply enter different values for the amount invested, cell C1 in the above spreadsheet, and the percentage rate, cell C2 in the above spreadsheet, and the spreadsheet automatically recalculates the amount for the end of each year. (If you do manage to create such a spreadsheet do not delete it as we will use it again in the next chapter when considering compound interest.)

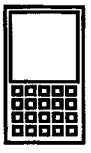
Example 1

Determine the interest paid on $2 500 invested at 8% per annum simple interest for 3 years. How much is the investment worth at the end of the three years?

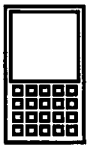
Interest each year is 8% of $2 500 = $2 500 × 0·08
= $200

Thus after 3 years the interest is $600.

Thus at the end of the three years the investment will be worth $2500 + $600
= $3100.



Alternatively we could use the built in capability of some calculators, or some computer or internet programs, to perform simple interest calculations.



The simple interest formula.

If we were to invest $P at R% per annum simple interest:

Interest received for one year = $\$P \times \frac{R}{100}$

After T years, the total interest = $\$P \times \frac{R}{100} \times T$.

Thus a *principal* of $P invested for T years at R% simple interest p.a. (per annum) earns interest of $I where

$$I = \frac{PRT}{100}$$

If instead we use R in decimal form, not as a percentage, the formula becomes

$$I = PRT$$

For example, if the interest rate were 6% the first formula would use R = 6 but the second formula would use R = 0·06.

Example 2

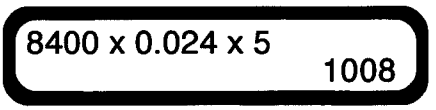
Determine the interest paid on \$8 400 invested at 2·4% per annum simple interest for 5 years.

Using $I = PRT$

$\text{Interest} = \$8\,400 \times 0{\cdot}024 \times 5$

$= \$1\,008$

The interest paid is \$1 008.



Example 3

Determine the value after two years of an investment of \$75 000 invested at 4·6% per annum simple interest.

Using $I = PRT$

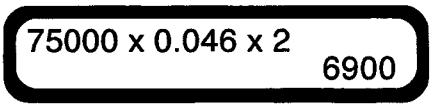
$\text{Interest} = \$75\,000 \times 0{\cdot}046 \times 2$

$= \$6\,900$

$\therefore$ Value after 2 years $= \$75\,000 + \$6\,900$

$= \$81\,900$

The value after two years is \$81 900.



Daily, monthly, quarterly and six monthly interest rates.

Suppose that T, the time a sum of money is invested for, is given as a number of days, or months, or quarters (a "quarter" is a quarter of a year, i.e. three months), or half year periods rather than as a number of years. If the annual interest rate is, for example, 6% then when using I = PRT:

With T in days we use $I = P \times \frac{0{\cdot}06}{365} \times T$

With T in months we use $I = P \times \frac{0{\cdot}06}{12} \times T$

With T in quarters we use $I = P \times \frac{0{\cdot}06}{4} \times T$

With T in 6 month periods we use $I = P \times \frac{0{\cdot}06}{2} \times T$

Note:

- For convenience, in this book, we will ignore leap years and assume that all years have 365 days.
- Some banking calculations, again for convenience, are based on a concept called *a banker's year.* This concept takes a year as being 360 days and consisting of twelve equal months each of thirty days.

Example 4

How much interest is due after 56 days for an investment of \$24 275 invested at 5% per annum. (Give your answer to the nearest cent.)

Using $I = PRT$

$$\text{Interest} = \$24\,275 \times \frac{0{\cdot}05}{365} \times 56$$

$$= \$186{\cdot}22 \text{ to the nearest cent.}$$

The interest due is \$186·22, to the nearest cent.

Example 5

How much interest is due after 10 months for an investment of \$125 000 invested at 4·7% per annum. (Give your answer to the nearest cent.)

Using $I = PRT$

$$\text{Interest} = \$125\,000 \times \frac{0{\cdot}047}{12} \times 10$$

$$= \$4895{\cdot}83 \text{ to the nearest cent..}$$

The interest due is \$4895·83, to the nearest cent.

Exercise 3A

1. Determine the interest paid on \$4 000 invested at 5% per annum simple interest for 2 years.

2. Determine the interest earned and the final value of the investment if \$500 is invested at 12% per annum simple interest for 6 years.

3. How much will an account be worth after 15 years if \$5000 is invested at 4·8% per annum simple interest?

4. A savings account paying five percent per annum simple interest is opened with an initial deposit of eight thousand seven hundred dollars. If no further deposits are made how much will this account be worth when it is closed seven years later?

5. Julie is left a sum of \$3450 from her late aunt's will. She decides to spend \$1000 of this amount on a holiday and invest the rest in an account paying 6·2% per annum simple interest.
   (a) How much will the account be worth three years later?
   (b) How much more would the account have been worth had Julie invested the whole \$3450 for the three years?

6. Jack invests \$5400 for three years in an account paying 4·5% per annum simple interest. How much more interest would he have received had he instead invested the same amount for the same time but in an account paying 5·4% per annum simple interest?

7. How much interest is earned when one million dollars is invested for one month in an account paying 5·4% per annum simple interest.

8. Suppose you were allowed to use one billion dollars (\$1000 000 000) for one day. How much interest would you receive if you invested the billion for the day in an account paying 6·2% per annum simple interest.

9. Shaq invests \$750 in an account paying 7·5% per annum simple interest for three quarters of a year. How much will the account be worth at the end of this time, to the nearest cent?

10. How much interest is earned when \$3500 is invested for 13 months in an account paying 9·8% per annum simple interest?

11. \$52 000 is invested for six months in an account paying 6·95% per annum simple interest. What will be the value of this account at the end of this time?

12. Penny has \$8300 to invest. The bank advises her that they have an account that normally pays 5·4% per annum simple interest but at the moment this account is offering an extra 1% per annum, to make the special offer interest rate of 6·4% per annum simple interest. If she invests the money for 200 days how much extra interest will the special offer give her in this time, over the normal rate?

13. How much interest accrues if \$17140 is invested for sixty days in an account paying 11·7% per annum simple interest.

The questions in the previous exercise involved simple interest rates that ranged from a low of 4·5% per annum to a high of 12% per annum. In reality, if you attempted to open a savings account today, is there likely to be that much variation in the various accounts that the various banks and financial institutions offer? Explain your answer.

Consider the table on the right showing the transactions and the balances for a bank savings account for the month of April and May.

Let us suppose that interest is paid on this account and the interest for April and May will be added on the first day of June. Hence interest for April will not itself earn interest in May.

Date	Details	Amount	Balance
01 April	Opening balance		\$250·00
07 April	Deposit	\$500·00	\$750·00
11 April	Cash withdrawal	\$350·00	\$400·00
21 April	Deposit	\$500·00	\$900·00
30 April	Final balance		\$900·00

01 May	Opening balance		\$900·00
05 May	Deposit	\$500·00	\$1400·00
13 May	Cash withdrawal	\$800.00	\$600·00
19 May	Deposit	\$500·00	\$1100·00
28 May	Cash withdrawal	\$430·00	\$670·00
31 May	Final balance		\$670·00

If interest is calculated at the rate of 6% per annum how much interest would the account earn for the month of April and for the month of May?

To be able to answer this question we need to know what method the bank uses to calculate the interest. Here we will consider two methods:

- Minimum monthly balance,

and

- Daily balance.

Minimum monthly balance.

From the account information given above we can see that during April the lowest balance was \$250·00 and the lowest balance during May was \$600·00. These lowest amounts are used to calculate the interest for each month in this *minimum monthly balance* method for determining interest.

For the 30 days of April, with a minimum balance of \$250·00 and an interest rate of 6% per annum:

$$\text{Interest for April} = \$250 \times \frac{0{\cdot}06}{365} \times 30$$
$$= \$1{\cdot}23 \text{ (rounded down)}$$

For the 31 days of May, with a minimum balance of \$600·00 and an interest rate of 6% per annum:

$$\text{Interest for May} = \$600 \times \frac{0{\cdot}06}{365} \times 31$$
$$= \$3{\cdot}05 \text{ (rounded down)}$$

Note • Rounding policy will depend on the particular rules for an account and would be decided upon by the bank or financial institution running the account.

Daily balance.

In this method interest is calculated on the account balance of each day. For the account details given above the account balance is \$250·00 on the 1st, 2nd, 3rd, 4th, 5th and 6th of April (i.e. 6 days), the balance changing to \$750 on the 7th April.

Balance of \$250·00 for 6 days:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

$$\text{Interest} = \$250{\cdot}00 \times \frac{0{\cdot}06}{365} \times 6 = \$0{\cdot}24 \quad \text{(rounded down)}$$

Balance of \$750·00 for 4 days:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

$$\text{Interest} = \$750{\cdot}00 \times \frac{0{\cdot}06}{365} \times 4 = \$0{\cdot}49 \quad \text{(rounded down)}$$

Balance of \$400·00 for 10 days:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

$$\text{Interest} = \$400{\cdot}00 \times \frac{0{\cdot}06}{365} \times 10 = \$0{\cdot}65 \quad \text{(rounded down)}$$

Balance of \$900·00 for 10 days:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

$$\text{Interest} = \$900{\cdot}00 \times \frac{0{\cdot}06}{365} \times 10 = \$1{\cdot}47 \quad \text{(rounded down)}$$

Total interest for April = \$0·24 + \$0·49 + \$0·65 + \$1·47
= \$2·85

Note • Again rounding policy will depend on the particular rules for an account. In the above calculations for the April interest, if all rounding was left to the end the total would be \$2·87 if rounded down, or \$2·88 to the nearest cent.

Similarly, to calculate the total interest for May:

Balance of \$900·00 for 4 days:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

$$\text{Interest} = \$900{\cdot}00 \times \frac{0{\cdot}06}{365} \times 4 = \$0{\cdot}59 \quad \text{(rounded down)}$$

Balance of \$1400·00 for 8 days:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

$$\text{Interest} = \$1400{\cdot}00 \times \frac{0{\cdot}06}{365} \times 8 = \$1{\cdot}84 \quad \text{(rounded down)}$$

Balance of \$600·00 for 6 days:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

$$\text{Interest} = \$600{\cdot}00 \times \frac{0{\cdot}06}{365} \times 6 = \$0{\cdot}59 \quad \text{(rounded down)}$$

Balance of \$1100·00 for 9 days:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

$$\text{Interest} = \$1100{\cdot}00 \times \frac{0{\cdot}06}{365} \times 9 = \$1{\cdot}62 \quad \text{(rounded down)}$$

Balance of \$670·00 for 4 days:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

$$\text{Interest} = \$670{\cdot}00 \times \frac{0{\cdot}06}{365} \times 4 = \$0{\cdot}44 \quad \text{(rounded down)}$$

Total interest for May = \$0·59 + \$1·84 + \$0·59 + \$1·62 + \$0·44
= \$5·08

Exercise 3B

Minimum monthly balance.

1. The account statement below is for the months of August (31 days), September (30 days) and October (31 days) for an account that pays interest of 6·2% per annum calculated monthly and based on the minimum monthly balance. Interest earned in these months is added to the account on the 1st of January.

Opening balance on the 1st August			**$245·56**
Date	**Deposit**	**Withdrawal**	**Balance**
05 August	$300·00		$545·56
21 August		$120·00	$425·56
03 September		$65·65	$359·91
10 September	$450·00		$809·91
25 September	$1250·00		$2059·91
06 October		$750·00	$1309·91
16 October	$125·00		$1434·91
Closing balance on the 31st October			**$1434·91**

Calculate the interest for each of August, September and October rounding your answer down to whole numbers of cents in each case.

2. The details below show the transactions that occur in an account that pays interest of 7·45% per annum calculated monthly and based on the minimum monthly balance. Interest earned in March (31 days), April (30 days) and May (31 days) will be added to the account on the 1st of July.

Date	**Credit**	**Debit**	**Balance**
15 February		$352·65	$1256·89
07 March	$2250·00		$3506·89
03 April		$1784·25	$1722·64
29 April	$2548·00		$4270·64
07 June		$3510·00	$760·64

Calculate the interest for each of March, April and May, rounding to the nearest cent.

3. The bank statement below shows all of the activity and balances for an account from 1st July one year until the 30th June of the following year. Interest is calculated monthly by applying one twelfth of the annual interest rate of 2·5% to the month's minimum balance and rounding the answer to the nearest cent. The annual interest is then added on the 1st July.
Calculate the interest earned for each of the twelve months July to June.

Date	**Activity**	**Amount**	**Credit**	**Debit**	**Balance**
01 Jul	Interest	$23·45	✓		$12 354·78
23 Oct	Cash deposit	$500·00	✓		$12 854·78
17 Mar	Cheque withdrawal	$7 354·60		✓	$5 500·18
08 Apr	Cash withdrawal	$1 000·00		✓	$4 500·18
26 Apr	Cheque withdrawal	$2 780·00		✓	$1 720·18
07 May	EFT deposit	$7562·75	✓		$9 282·93

Daily balance.

4. The table on the right shows the transactions and the balances for a bank account for the months of April and May.
Let us suppose that interest is paid on this account and the interest for April, May and June will be added on the first day of July.
If interest is calculated at the rate of 4% per annum based on the daily balance how much interest would the account earn for the month of April and for the month of May? (Calculate intermediate values rounded to 4 decimal places until totaling the monthly interest, then round this monthly total to the nearest cent.)

Date	Details	Amount	Balance
01 April	Opening balance		$1352·68
11 April	Deposit	$750·00	$2102·68
22 April	Chq withdrawal	$265·75	$1836·93
25 April	Deposit	$750·00	$2586·93
30 April	Final balance		$2586·93
01 May	Opening balance		$2586·93
09 May	Deposit	$750·00	$3336·93
11 May	Cash withdrawal	$675.00	$2661·93
22 May	Chq deposit	$375·25	$3037·18
23 May	Deposit	$750·00	$3787·18
25 May	Cash withdrawal	$430·00	$3357·18
31 May	Final balance		$3357·18

5. The details below show the transactions that occur in an account that pays interest of 3·25% per annum calculated on the daily balance. Interest earned for the first six months of the year is added on the 1st of July. Also shown is the balance on the 1st January.
Calculate the interest earned for the period 1st January to 31st March calculating intermediate values rounded to 4 decimal places and then giving the total rounded to the nearest cent. (Assume the year involved is not a leap year.)

Date	Credit	Debit	Balance
01 Jan	-	-	$1256·43
23 Jan	$1250·00		
03 Feb		$356·54	
14 Feb	$876·50		
07 Mar		$782·64	
23 Mar		$1254·80	
14 Apr	$525·90		

Borrowing money.

So far this chapter has been involved with earning interest on invested money. However not everyone has money to invest. Sometimes we need to borrow money from a bank or financial institution and in such cases we are charged interest on the borrowed money.

Banks make profits by charging a higher interest rate on the money they lend out than they pay on money that is invested with them.

The remainder of this chapter considers simple interest charged on loans, rather than interest paid out on invested funds.

Example 6

How much needs to be repaid after three years to repay a loan of \$8000 at 5·4% per annum simple interest.

Using $I = PRT$

$\text{Interest} = \$8\,000 \times 0{\cdot}054 \times 3$

$= \$1296.$

Hence the amount owed $= \$8000 + \$1296$

$= \$9296$

\$9296 needs to be repaid after three years.

Exercise 3C

1. How much interest is charged on a loan of \$25 000 borrowed for 2 years with simple interest charged at 7% per annum.
2. How much interest is charged on a loan of \$8 750 borrowed for 5 years with simple interest charged at 8·4% per annum.
3. How much interest is charged on a loan of \$2 500 borrowed for 17 months with simple interest charged at 4·75% per annum.
4. How much interest is charged on a loan of \$6500 borrowed for 150 days with simple interest charged at 18% per annum?
5. How much needs to be repaid after 4 years to repay a loan of \$7500 at 8·2% per annum simple interest
6. How much needs to be repaid after five years to repay a loan of \$14500 at 12·5% per annum simple interest
7. How much is owed after three years if \$250 is borrowed at 18% per annum simple interest with no repayments being made before the end of the three years?
8. Tarni borrows \$7600 to purchase a car. She agrees to repay the loan in full, plus simple interest charged at the rate of 8·5% per annum, after three years. How much will she have to pay at the end of the three years to clear the loan?
9. Frank borrows \$8000 for three years with the interest fixed at 10% per annum simple interest for the three years. One year into this loan he finds he needs to borrow a further \$2500 but now the simple interest rate is 12·5%. How much does he owe altogether on these two loans at the end of the three years given that he makes no repayments during the three years?
10. Ali borrows \$5000 at a simple interest charge of 8% per annum. After 21 months he renegotiates the loan by paying off \$3000 and having the remaining principal plus interest moved into a new loan account that charges simple interest at the rate of 7·5% per annum. If he wishes to pay off what he owes on this new account two years later how much will he have to pay to do so?

Miscellaneous Exercise Three.

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the preliminary section at the beginning of the book.

1. Round each of the following to 2 decimal places.
   (a) 35·327 (b) 25·824 (c) 56·97193 (d) 27·259812

2. Copy and complete the following table:

	What it cost.	What it was sold for.	Profit as percentage of cost.
(a)	\$200	\$250	
(b)	\$450		20%
(c)		\$2 310	40%

	What it cost.	What it was sold for.	Loss as percentage of cost.
(d)	\$200	\$190	
(e)	\$8 500		2%
(f)		\$93 600	40%

3. (a) Find 25% of \$500.
   (b) Increase \$500 by 25%.
   (c) \$500 is 25% of an amount. Find the amount.
   (d) After an increase of 25% an amount becomes \$500. What was the amount before the increase?

4. State the "odd one out" A, B or C, in each of the following.
   (a) A: 0·2 B: $\frac{1}{2}$ C: 20%
   (b) A: 2·5% B: $\frac{1}{4}$ C: 0·025
   (c) A: 0·34 B: 75% C: $\frac{3}{4}$
   (d) A: 0·05 B: 5% C: $\frac{1}{25}$

5. What is the value at the end of five years of an initial investment of \$5800 in an account paying a simple interest rate of 4·4% per annum?

6. Which pays more interest:
   A: \$5000 invested for 4 years in an account paying 7% per annum simple interest.
   Or
   B: \$7000 invested for 5 years in an account paying 4% per annum simple interest.

7. $\boxed{A = \pi rs + \pi r^2}$
   Find A given $r = 5$ and $s = 8$. (Answer correct to one decimal place.)

8. Motor accident investigators may measure the length of a skid mark left by a braking vehicle. If a vehicle skids s metres in coming to rest then the formula

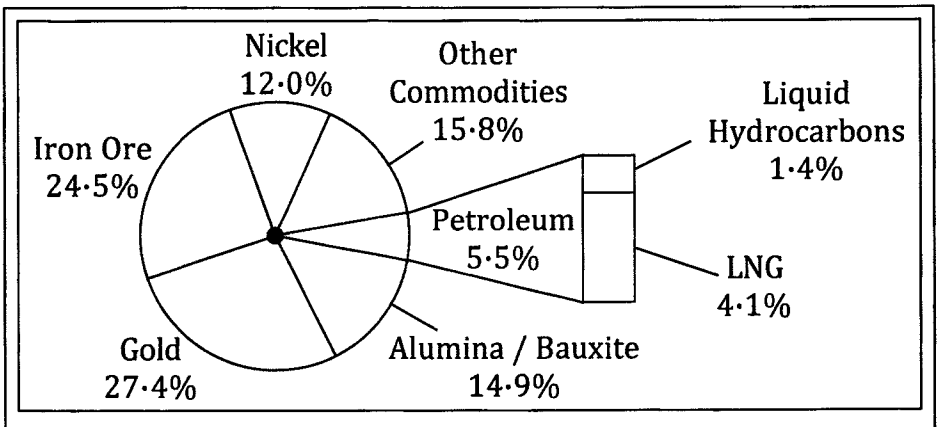
$$v = 4\sqrt{10s}$$

can be used to give an estimate for v, the speed of the vehicle in km/h at the commencement of the skid.

(a) Estimate the speed of a vehicle that left a skid mark of 22 metres in coming to rest.

(b) Crash investigators measure a skid to be approximately 50 metres. The driver of the vehicle causing this skid mark claimed that he had not been exceeding the speed limit of 110 km/h. Does the formula support his claim or not?

9. The pie graph below shows how the approximately 47000 people employed in the resources sector in Western Australia in a particular year were distributed across the commodities.



[Data Source: The Chamber of Minerals & Energy .]

(a) Notice that the pie graph percentages add up to 100·1%:

$$24{\cdot}5 + 12{\cdot}0 + 15{\cdot}8 + 5{\cdot}5 + 14{\cdot}9 + 27{\cdot}4 = 100{\cdot}1$$

Surely the total should be 100%. Explain how the "extra" 0·1% could occur.

(b) How many of the people employed in the resources sector in Western Australia for this particular year were employed in (i) Gold, (ii) LNG?

(c) Twelve years later the number of people employed in the resources sector in Western Australia had risen to approximately 70000. Predict the number employed in the Nickel sector at this later time, explaining any assumptions you are making.

10. A second hand car salesperson purchases a fleet of 10 used cars for $12 000 each. He manages to sell 7 of them at a profit of 16% each. He wishes to sell the remaining 3 cars for the same price as each other and at a price that will give him a 15% profit overall for the ten cars. What price should he sell each of the remaining cars for?